斜面安定解析のための極限平衡分割法の比較と評価：モーメント平衡式の改良と定常浸透状態における水中重量法

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Review and Comparison of Limit Equilibrium Methods of Slices for Slope Stability Analysis

― Revised Equation of Moment Equilibrium and Buoyancy under Steady Seepage Flow Condition ―

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Contents

Ⅰ Introduction

Around 40% of farmland in Japan is located in the sloped land of mountainous and semi-mountainous areas. In the landslide areas of the Tertiary or Schist layers, much farmland has been historically developed since ancient times, because there have been fertile clayey soils in the crushed zone of these layers. Also, it has been easy for farmers not only to get irrigation water in many places but also to cultivate the land in sloped areas, even though landslide disasters have often occurred. Therefore, slope stability is an important subject for farmland conservation and reclamation in Japan. The author investigated slope stability analysis for the prevention and control of landslides, and/or for the design of embankment construction in sloped areas.

A large number of limit equilibrium methods of slices have been proposed and utilized for slope stability analysis. The Ordinary method of slices and the Simplified Bishop method (Bishop, 1955) are commonly used for slip circles. The Fellenius method is widely used for the analysis of landslides in Japan. The Janbu method (Janbu, 1954) is also

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well known and is often used for the analysis of slopes with a slip surface of general shape. The Nonveiller method (Nonveiller, 1965), which is introduced from the moment equilibrium and extended to slip surfaces of general shape, is also convenient for the analysis of landslides.

On the other hand, a general limit equilibrium method of slices was developed by Morgenstern, N.R. and Price, V.E.(1965). This method satisfies force and moment equilibrium conditions and is often used for case histories of landslides all over the world. The Spencer method (Spencer, 1967), which assumes parallel inter-slice forces, is one of the general limit equilibrium methods of slices for a slip circle. This method can be extended easily to slip surfaces of general shape by the modified angle of inter-slice forces (Spencer, 1969). Spencer succeedingly published another general limit equilibrium method of slices for slip surfaces of general shape (Spencer, 1973), which assumes each variable angle of inter-slice force between slices. These methods, which assume inter-slice force or factors, such as $\lambda \cdot f(x)$ of the Morgenstern-Price method or θ_i , Q of the Spencer methods, are practical and useful in converging computations. Fredlund and Krahn (1981) compared some limit equilibrium methods of slices and explained clearly the relationship between these methods.

In this paper, some limit equilibrium methods of slices are reviewed. Also, a general limit equilibrium method of slices is introduced (Furuya, 1986) and is examined by comparing with the Morgenstern & Price and the Spencer methods. The moment equation of this method is derived from the equilibrium of resultant vertical and horizontal inter-slice forces. Methods to calculate pore water pressure are also discussed as related to buoyancy under steady seepage flow conditions (Furuya, 1985).

In addition, this paper was rewritten from「Examination of Slope Stability Methods of Analysis(in Japanese)」reported in the symposium of the Japan Landslide Society in 1998.

Ⅱ Forces acting on a slice

Figure 1 shows the forces acting on a slice of a slip surface and Fig. 2 shows a polygon of those forces. The method of calculation for pore water pressure by u, P_w, P_{Si}, P_{Si+1}, is defined as the total pore water pressure method (Yamagami and Ueda, 1982) in chapters 3, 4 and 5 of this paper.

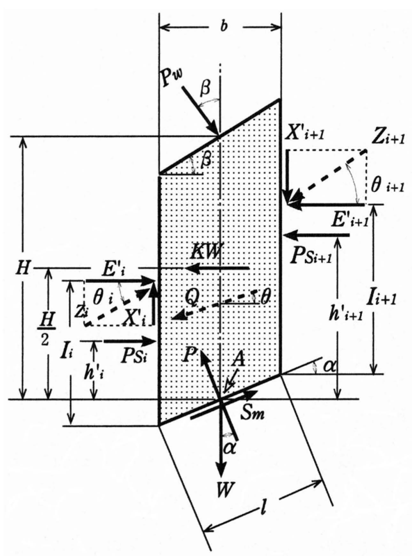


Fig. 1 Forces acting on a slice of a slip surface.

Notation

W = total weight of slice of width b and height H

α = angle of base of slice

β = angle of upper of slice

U = force due to pore water pressure on base of slice

u = pore water pressure, l = length of base of slice, $U = ul$
P = total force normal to base of slice
p' = effective force normal to base of slice, $P = p' + ul$
c' = cohesion with respect to effective stress
c'_m = mobilized cohesion: c'/F, F = factor of safety
ϕ' = angle of shearing resistance with respect to effective stress
ϕ'_m = mobilized angle of shearing resistance. $\tan\phi'_m = (\tan\phi')/F$
S = total shear force available
S_m = mobilized shear force on base of slice

$$S_m = \{ c'l + (P - ul)\tan\phi' \} / F$$

P_w = force due to water pressure on upper part of slice
P_{si}, P_{si+1} = forces due to water pressure on both sides of slice
E'_i, E'_{i+1} = horizontal inter-slice forces with respect to effective stress on both sides of slice
X'_i, X'_{i+1} = vertical inter-slice forces with respect to effective stress on both sides of slice
Z_i, Z_{i+1} = resultant inter-slice forces of E'_i, X'_i and E'_{i+1}, X'_{i+1}
θ_i, θ_{i+1} = angle determining slope of inter-slice force Z_i, Z_{i+1}
$X'_i = E'_i \tan\theta_i$, $X'_{i+1} = E'_{i+1}\tan\theta_{i+1}$
Q = resultant inter-slice forces of Z_i and Z_{i+1}
θ = angle of inter-slice forces Q
K = Seismic coefficient to account for a dynamic horizontal force. Acting point is assumed at point $H/2$

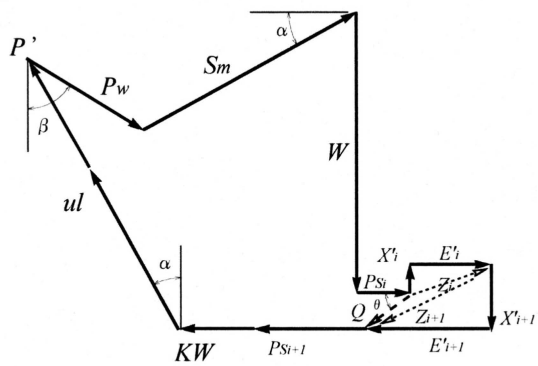


Fig. 2 Polygon of forces acting on a slice.

Ⅲ Force and moment equilibrium conditions and methods of slope stability analysis

All limit equilibrium methods of slices for slope stability analysis are introduced from force and/or moment equilibrium conditions based on assumptions concerned with inter-slice force, because the number of equations does not correspond to the unknowns and assumptions.

Equilibrium conditions are as follows:

① Force equilibrium in the vertical direction (or in the direction perpendicular to base of slice)
② Force equilibrium in the horizontal direction (or in the direction parallel to base of slice)
③ Moment equilibrium about a common point

1 The Ordinary or The Fellenius method

The following equation of the Ordinary or Fellenius method is widely used for the computation of factors of safety in the field of practical civil engineering in Japan.

$$F = \frac{\sum\{ c'l + (W\cos\alpha - ul)\tan\phi' \}}{\sum W\sin\alpha} \qquad (1)$$

Forces acting on a slice are shown in Figs. 1 and 2. The computation considers the pore water pressures acting on the upper, both sides and the base of the slice, and seismic coefficient. The normal force on the base of the slice is derived from the summation of forces perpendicular to the base of the slice (Fig. 2).

$$W\cos\alpha + (P_{Si} - P_{Si+1} - KW)\sin\alpha - Q\sin(\alpha - \theta) - p' - ul + P_w\cos(\beta - \alpha) = 0 \quad \cdots\cdots (2)$$

The inclination of resultant inter-slice force is assumed to be parallel to the base of the slice in the Ordinary method, thus, $\sin(\alpha - \theta)$ equals zero. Therefore, the following equation is derived.

$$p' = W\cos\alpha + (P_{Si} - P_{Si+1} - KW)\sin\alpha - ul + P_w\cos(\beta - \alpha) \quad \cdots\cdots (3)$$

The following equation is also derived from the summation of forces parallel to the base of the slice (Fig. 2).

$$P_w\sin(\beta - \alpha) + S_m = W\sin\alpha + Q\cos(\alpha - \theta) - (P_{Si} - P_{Si+1} - KW)\cos\alpha \quad \cdots\cdots (4)$$

Substituting (3) into (4), the equation for factor of safety is derived as follows:

$$F = \frac{\sum\{c'l + ([\boldsymbol{K}] - \mathrm{ul} - [\boldsymbol{J}]\sin\alpha)\tan\phi'\}}{\sum\{W\sin\alpha + [\boldsymbol{J}]\cos\alpha - P_w\sin(\beta - \alpha) + Q\}} \quad \cdots\cdots (5)$$

where $[\boldsymbol{K}] = W\cos\alpha + P_w\cos(\beta - \alpha)$

$[\boldsymbol{J}] = KW + P_{Si} - P_{Si+1}$

2 The Janbu's method

Two methods, the Simplified and the Rigorous methods, are included in the Janbu's methods (Janbu, 1954). The normal force p' on the base of each slice is derived from the summation of vertical forces (Fig. 2). The computation includes the pore water pressures acting on the upper, both sides and the base of the slice, and seismic coefficient.

$$W + X'_{i+1} - X'_i + P_w\cos\beta = (p' + ul)\cos\alpha + S_m\sin\alpha \quad \cdots\cdots (6)$$

From equation (6),

$$p' = \frac{[M] - c'_m\, l\sin\alpha}{m\alpha} \quad \cdots\cdots (7)$$

where $[M] = W + X'_{i+1} - X'_i + P_w\cos\beta - ul\cos\alpha$

and $m_\alpha = \cos\alpha + \tan\phi'_m\sin\alpha$

The equation for factor of safety is derived as follows from the summation of horizontal forces (Fig. 2).

$$\mathrm{F} = \frac{\sum\{c'l + p'\tan\phi'\}\cos\alpha}{\sum(p' + ul)\sin\alpha + \sum KW - \sum[W_a]} \quad \cdots\cdots (8)$$

where $[W_a] = P_w\sin\beta + (P_{Si} - P_{Si+1} + E_i - E_{i+1})$

The summation of inter-slice force must cancel in equation (8) of the simplified Janbu method and the factor of safety can be computed by iteration. Correction factor f_0 is prepared as relating to cohesion, angle of internal friction, and shape of failure surface.

In the Janbu's rigorous method, the acting point of inter-slice force is assumed to be *1/3 H* from the bottom of the slice corner. X_i and X_{i+1} are evaluated from the conditions of moment equilibrium in each slice about the middle point of the base in order to calculate p'.

3 The Simplified Bishop and The Nonveiller method

The Nonveiller method is extended to general slip surfaces from the simplified Bishop for slip circles (Nonveiller, 1965). The normal force p' on the base of the slice is also derived from the summation of vertical forces in the force polygon of Fig. 2 [Equation (7)]. Inter-slice forces must cancel in total sliding mass. Therefore, from the summation of moment equilibrium of forces acting on each slice about a common point O_c (Fig. 3), the following equation is derived:

$$\Sigma W \cdot x = \Sigma S \cdot a + \Sigma P \cdot f - \Sigma KW(y + H/2) - \Sigma P_w \cos\beta \cdot x + \Sigma P_w \sin\beta \cdot y \quad \cdots\cdots (9)$$

From equation (9), we obtain the equation for the factor of safety as follows:

$$F = \frac{\Sigma(c'l + p' \tan\phi') \cdot a}{\Sigma W \cdot x - \Sigma P \cdot f + \Sigma KW(y + H/2) + [L]} \quad \cdots\cdots (10)$$

where $[L] = \Sigma P_w \cos\beta \cdot x - \Sigma P_w \sin\beta \cdot y$

Equation (10) becomes the simplified Bishop method (Bishop, 1955), because f equals zero in the slip circle and becomes the Nonveiller method in the slip surfaces of general shape. Inter-slice forces X'_{i+1}, X'_i are ignored in both methods. In case that the inter-slice forces are considered, equation (10) is called the Bishop's rigorous method in the slip circle. The factor of safety can be computed by iteration.

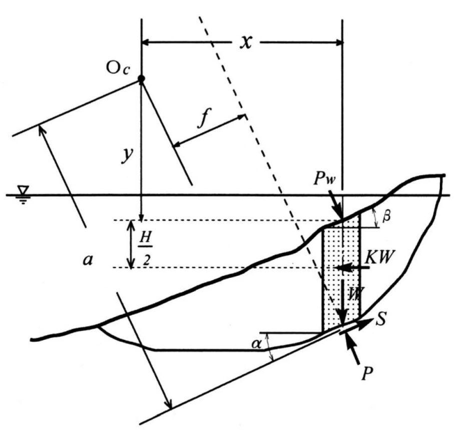


Fig. 3 Moment of external forces acting on each slice about a common point O_c.

4 The Morgenstern-Price method

In the Morgenstern-Price method (Morgenstern & Price, 1965), a relationship between vertical and horizontal inter-slice forces is assumed as follows:

$$X = \lambda \cdot f(x) \cdot E' \quad \cdots\cdots (11)$$

where X denotes the vertical shear force on the side of the slice,
λ is a parameter,
and $f(x)$ is the factor of inter-slice force.

From the equilibrium conditions acting on an infinitesimal slice (Fig. 4), we obtain:

$$E = \frac{1}{L+Kx}\left[E_i L + \frac{Nx^2}{2} + Px\right] \quad \cdots\cdots (12)$$

where

$$K = \lambda k \left\{ \frac{\tan \phi'}{F} + A \right\}$$

$$L = \lambda m \left\{ \frac{\tan \phi'}{F} + A \right\} + 1 - A \frac{\tan \phi'}{F}$$

$$N = P \left[\frac{\tan \phi'}{F} + A - r_u(1+A^2) \frac{\tan \phi'}{F} \right]$$

$$P = \frac{c'}{F}(1+A^2) + q \left[\frac{\tan \phi'}{F} + A - r_u(1+A^2) \frac{\tan \phi'}{F} \right]$$

$$y = Ax + B$$

$$\frac{dW}{dx} = px + q$$

$f = kx + m$, the function f is defined by equation (11) depends linearly on x.

E_0 equals zero at the beginning of the slip surface in the usual case. The value of E_n at the end of a slip surface is determined by integration across each slice. E_n is usually zero from the boundary conditions.

From the moment equilibrium about the mid point of the infinitesimal slice, after simplifying and proceeding to the limit as $dx \to 0$, the following equation is derived:

$$X = \frac{d}{dx}(E' \cdot y'_t) - y\frac{dE'}{dx} + \frac{d}{dx}(P_w \cdot h) - y\frac{dP_w}{dx} \quad \cdots\cdots (13)$$

By integrating equation (13), we obtain:

$$M = E(y_t - y) = \int_{x0}^{x} \left(X - E\frac{dy}{dx}\right)dx \quad \cdots\cdots (14)$$

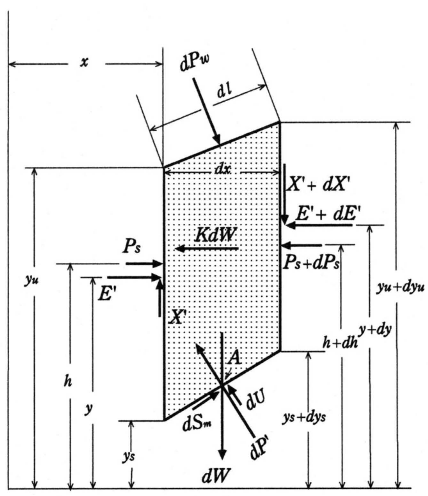


Fig. 4 Forces acting on an infinitesimal slice.

Since moment equals zero at the end of the slip surface in general, $M_n = 0$ when moment equilibrium is satisfied in a slip surface. The factor of safety is computed from equations (12) and (14) by the Newton-Raphson method in assuming $f(x)$ values. The validity of the results needs to be examined from the magnitude and distribution of inter-slice force and it's acting point.

5 The Spencer methods

Spencer proposed two methods of slices (Spencer, 1967, 1973). The resultant inter-slice force of Spencer method (1967) for slip circle is expressed as Q, of which angle is assumed parallel in all slices. This method can be extended to general shapes by modifying angles of the inter-slice force Q (Spencer, 1969). In the Spencer method (1973), inter-slice force X', E' is expressed as resultant force Z and angle of inter-slice force Z is defined as θ_i. The method in 1973 is described in this section.

From two equations of vertical and horizontal force equilibrium conditions in Fig. 2, an expression between Z_i and Z_{i+1} is obtained as follows:

$$Z_{i+1} = Z_i \frac{[G]}{[F]} + \frac{[B]+[C]+[D]}{[F]} \quad \cdots\cdots (15)$$

where $[G] = \cos(\alpha - \theta_i) + \tan\phi'_m \sin(\alpha - \theta_i)$

$[B] = \tan\phi'_m (W\cos\alpha - ul) + c'_m b\sec\alpha - W\sin\alpha$

$[C] = (P_{Si} - P_{Si+1} - KW) \cdot (\cos\alpha + \tan\phi'_m \sin\alpha)$

$[D] = \{\sin(\beta - \alpha) + \tan\phi'_m \cos(\beta - \alpha)\} \cdot P_w$

$[F] = \cos(\alpha - \theta_{i+1}) + \tan\phi'_m \sin(\alpha - \theta_{i+1})$

For the first slice, Z_0 is taken as zero and the expressions give the value of the next Z_{i+1}, step by step, finally giving Z_n. When the force equilibrium is satisfied in a sliding mass, inter-slice force at the end of the last slice becomes zero.

$$Z_n = 0 \quad \cdots\cdots (16)$$

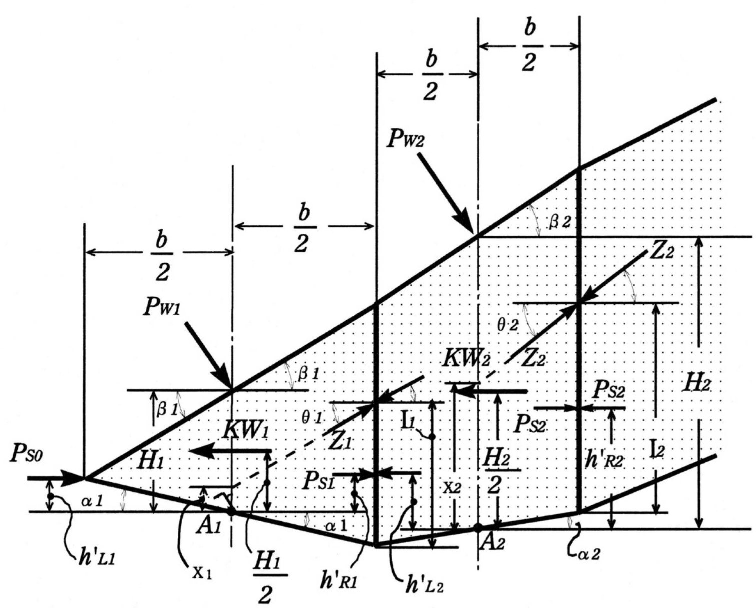


Fig. 5 Moment of forces about point A_1 .

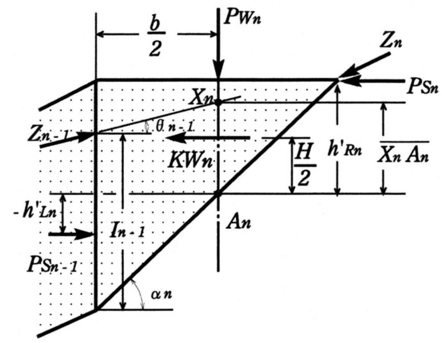


Fig. 6 Moment of forces about point A_n .

From the moment equilibrium about the middle of the base of the first slice (A_1, Fig. 5), the vertical distance X_1 from A_1 of inter-slice force Z_1 is derived as follows:

$$X_1 = \frac{P_{w1} H_1 \sin\beta_1 - KW_1 \cdot H/2 + P_{S0} h'_{L1} - P_{S1} h'_{R1}}{Z_1 \cos\theta_1} \quad (17)$$

therefore,

$$I_1 = b_1 \tan\theta_1/2 - b_1 \tan(-\alpha_1)/2 + \frac{P_{w1} H_1 \sin\beta_1 - KW_1 \cdot H_1/2 + P_{S0} h'_{L1} - P_{S1} h'_{R1}}{Z_1 \cos\theta_1} \quad (18)$$

By step by step computations of I_2, I_3, $\cdots$, I_n is derived from deduction as follows:

$$I_n = b_n (\tan\theta_n - \tan\alpha_n)/2 + \frac{1}{Z_n \cos\theta_n} \sum_{j=1}^{n} [J] \quad (19)$$

where $J = Z_{j-1} \{\sin\theta_{j-1} (b_j + b_{j-1}) - \cos\theta_{j-1} (b_j \tan\alpha_j + b_{j-1} \tan\alpha_{j-1})\}/2 + P_{wj} H_j \sin\beta_j$

$- KW_j H_j/2 + P_{Sj-1} h'_{Lj} - P_{Sj} h'_{Rj}$

From the moment equilibrium about the middle point (A_n) of the base of the last slice (Fig. 6), the following equation is obtained.

$$\overline{X_nA_n} \cos\theta_{n-1} Z_{n-1} + P_{wn} H_n \sin\beta_n/2 - H_n KW_n + P_{Sn-1}(-h'_{Ln}) - P_{Sn} h'_{Rn} = 0 \quad (20)$$

Therefore,

$$\overline{X_nA_n} = \frac{H_n KW_n/2 - P_{wn} H_n \sin\beta_n + [PH]}{Z_{n-1} \cos\theta_{n-1}} \quad (21)$$

where $[PH] = P_{Sn} h'_{Rn} - P_{Sn-1}(-h'_{Ln})$

$\overline{X_nA_n}$ is also derived using equation (19) in the last slice as follows:

$$\overline{X_nA_n} = I_{n-1} + b_n (\tan\theta_{n-1} - \tan\alpha_n)/2$$

$$= b_{n-1} (\tan\theta_{n-1} - \tan\alpha_{n-1})/2 + \frac{1}{Z_{n-1} \cos\theta_{n-1}} \sum_{j=1}^{n-1} [J] + b_n (\tan\theta_{n-1} - \tan\alpha_n)/2 \quad (22)$$

where

$$\sum_{j=1}^{n-1}[J]=\sum_{j=1}^{n}[J]-Z_{n-1}\{\sin\theta_{n-1}(b_n+b_{n-1})-\cos\theta_{n-1}\quad(b_n\tan\alpha_n+b_{n-1}\tan\alpha_{n-1})\}/2$$

$$-P_{wn}H_n\sin\beta+H_nW_n/2-P_{Sn-1}(-h'_{Ln})+P_{Sn}h'_{Rn}\quad\cdots\cdots\quad(23)$$

From equations (21), (22), the following expression is obtained for the moment equilibrium in a sliding mass.

$$\sum_{j=1}^{n-1}[J]=0\quad\cdots\cdots\quad(24)$$

The factor of safety is calculated from equations (16) and (24) by the Newton-Raphson method. The validity of the results needs to be examined from the magnitude and distribution of the inter-slice force and it's acting point.

6 The proposed method

This method (Furuya, 1986) was proposed assuming the parameter θ_i of an angle of inter-slice force similar to the Spencer method (1973). It generalizes a common point of moment equilibrium different from the first point of slip surfaces. From the vertical and horizontal force equilibrium (Fig. 2), the following expression concerning E'_{i+1}, E'_i is obtained, assuming the relationship between the vertical and horizontal inter-slice forces, $X'_i=E'_i\tan\theta_i$.

$$E'_{i+1}=\frac{[A]}{[E]}E'_i+\frac{[B]+[C]+[D]}{[E]}\quad\cdots\cdots\quad(25)$$

where $[A]=\tan\theta_i\sin\alpha+\cos\alpha-\tan\phi'_m(\tan\theta_i\cos\alpha-\sin\alpha)$

$[B]=\tan\phi'_m(W\cos\alpha-ul)+c'_m b\sec\alpha-W\sin\alpha$

$[C]=(P_{Si}-P_{Si+1}-KW)\cdot(\cos\alpha+\tan\phi'_m\sin\alpha)$

$[D]=\{\sin(\beta-\alpha)+\tan\phi'_m\cos(\beta-\alpha)\}P_w$

$[E]=\tan\theta_{i+1}\sin\alpha+\cos\alpha-\tan\phi'_m(\tan\theta_{i+1}\cos\alpha-\sin\alpha)$

E'_0 is taken as zero for the first slice and the expressions give the value of the next E'_{i+1}, step by step, finally giving E'_n. When the force equilibrium is satisfied in a sliding mass, inter-slice force at the end of the last slice becomes zero.

$$E'_n=0\quad\cdots\cdots\quad(26)$$

Considering the moment equilibrium condition, the resultant force of E'_i and E'_{i+1} is expressed as ΔE and the resultant force of X'_i and X'_{i+1} is expressed as ΔX. The moment of inter-slice forces, ΔX and ΔE, in each slice about a common point O_c (Fig. 7) is expressed as follows:

$$M_i=x_{0i}\,\Delta X+(y_{0i}-h)\,\Delta E=x_{0i}\,\Delta X+y_{0i}\,\Delta E-h\,\Delta E\quad\cdots\cdots\quad(27)$$

On the other hand, the summation of the moment of forces in each slice about the mid point of the base must also cancel. Then, we have the following equation:

$$-h\Delta E+m_i=0\quad\cdots\cdots\quad(28)$$

where $m_i = H_i P_{wi} \sin\beta_i - H_i KW_i / 2 + P_{si} h'_i - P_{si+1} h'_{i+1}$

Substituting (28), and $X'_i = E'_i \tan\theta_i$, $X'_{i+1} = E'_{i+1} \tan\theta_{i+1}$ into (27), we obtain the following equation:

$$M_i = x_{oi} (X'_{i+1} - X'_i) + y_{oi} (E'_{i+1} - E'_i) - m_i$$

$$= x_{oi} (E'_{i+1} \tan\theta_{i+1} - E'_i \tan\theta_i) + y_{oi} (E'_{i+1} - E'_i) - m_i$$

$$= (x_{oi} \tan\theta_{i+1} + y_{oi}) E'_{i+1} - (x_{oi} \tan\theta_i + y_{oi}) E'_i - m_i \quad \cdots\cdots (29)$$

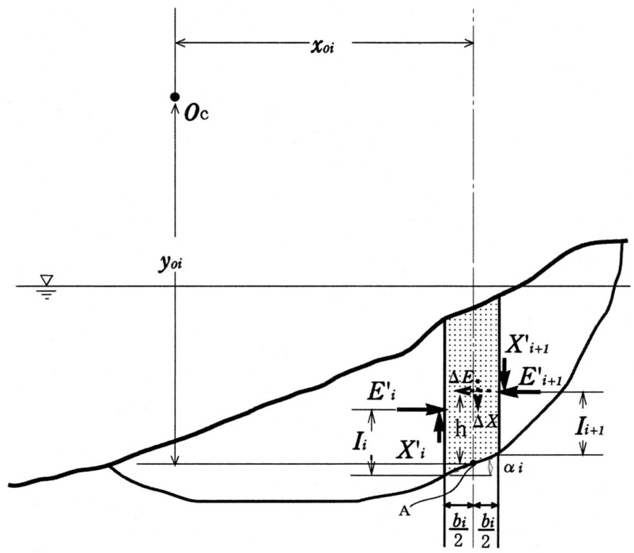


Fig. 7 Moment of the inter-slice forces ΔX, ΔE about a common point O_c.

When the moment equilibrium condition about a common point O_c is satisfied in a sliding mass, the summation of moment of inter-slice forces must cancel in the sliding mass. Therefore, we have the following equation:

$$\sum_{i=1}^{n} [M_i] = 0 \quad \cdots\cdots (30)$$

The factor of safety is computed from equations (26) and (30) by the Newton-Raphson method or linear reverse interpolation method. The validity of the results needs to be examined from the magnitude and distribution of inter-slice force and it's acting point.

7 Relationship between the proposed method and the Spencer method

In this section, the moment equilibrium equations of the proposed method and the Spencer method (1973) are discussed. In the moment equation of inter-slice forces about the common point O_c by equation (29), E'_i, x_{oi} and y_{oi} can be replaced by Z_i, x_{zi} and y_{zi} respectively. Then, the moment equation of inter-slice forces by equation (29) about the common point O_z, the first point in a sliding mass (Fig.8) , is expressed as follows:

$$M_i = (x_{zi} \cdot \sin\theta_i - y_{zi} \cdot \cos\theta_i) \cdot Z_i - (x_{zi} \cdot \sin\theta_{i-1} - y_{zi} \cdot \cos\theta_{i-1}) \cdot Z_{i-1} - m_i \quad \cdots\cdots (31)$$

Calculating x_{zi} and y_{zi} by b_i, $b_i \cdot \tan\alpha_i$ sequentially,

$$M_1 = (b_1/2 \cdot \sin\theta_1) - b_1/2 \cdot \tan\alpha_1 \cdot \cos\theta_1 \cdot Z_1 - m_1 \quad \cdots\cdots (32)$$

$$M_2 = \{(b_1 + b_2/2)\cdot\sin\theta_2 - (b_1\cdot\tan\alpha_1 + b_2/2\cdot\tan\alpha_2)\cdot\cos\theta_2\}\cdot Z_2 - \{(b_1+b_2/2)\cdot\sin\theta_2 - (b_1\cdot\tan\alpha_1 + b_2/2\cdot\tan\alpha_2)\cdot\cos\theta_2\}\cdot Z_1 - m_2 \quad\cdots\cdots (33)$$

$$M_3 = \{(b_1 + b_2 + b_3/2)\cdot\sin\theta_3 - (b_1\tan\alpha_1 + b_2\tan\alpha_2 + b_3/2\cdot\tan\alpha_3)\cdot\cos\theta_3\}\cdot Z_3 - \{(b_1 + b_2 + b_3/2)\cdot\sin\theta_3 - (b_1\tan\alpha_1 + b_2\tan\alpha_2 + b_3/2\cdot\tan\alpha_3)\cdot\cos\theta_3\}\cdot Z_2 - m_3 \quad\cdots\cdots (34)$$

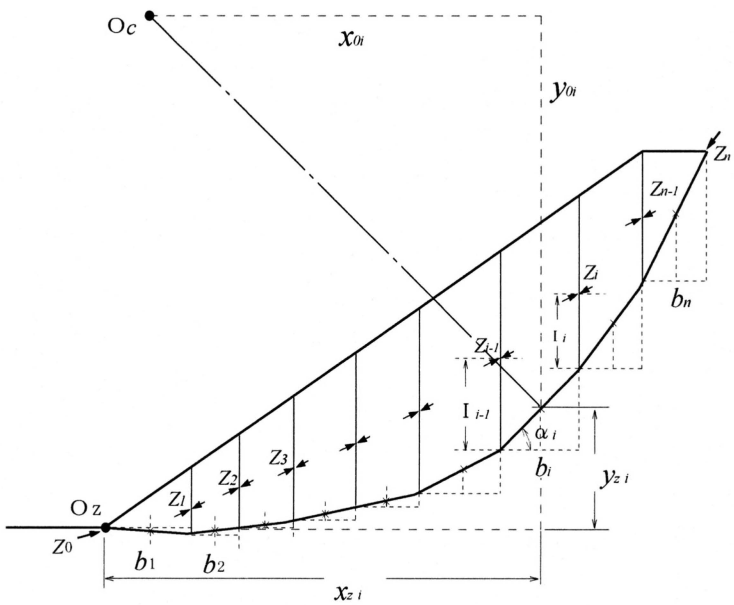


Fig. 8 The arm length of moment for inter-slice force in a sliding mass.

$$M_n = \{(b_1 + \cdots + b_{n-1} + b_n/2)\cdot\sin\theta_n - (b_1\tan\alpha_1 + \cdots + b_{n-1}\tan\alpha_{n-1} + b_n/2\cdot\tan\alpha_n)\cdot\cos\theta_n\}\cdot Z_n - \{(b_1 + \cdots + b_{n-1} + b_n/2)\cdot\sin\theta_{n-1} - (b_1\tan\alpha_1 + \cdots + b_{n-1}\tan\alpha_{n-1} + b_n/2\cdot\tan\alpha_n)\cdot\cos\theta_{n-1}\}\cdot Z_{n-1} - m_n \quad\cdots\cdots (35)$$

The moment equation in a sliding mass is derived as follows:

$$\sum_{i=1}^{n}[M_i] = -Z_1\cdot\{\sin\theta_1\cdot(b_1+b_2) - \cos\theta_1\cdot(b_1\tan\alpha_1 + b_2\tan\alpha_2)\}/2$$
$$-Z_2\cdot\{\sin\theta_2\cdot(b_2+b_3) - \cos\theta_2\cdot(b_2\tan\alpha_2 + b_3\tan\alpha_3)\}/2$$
$$\vdots$$
$$-Z_{n-1}\cdot\{\sin\theta_{n-1}\cdot(b_{n-1}+b_n) - \cos\theta_{n-1}\cdot(b_{n-1}\tan\alpha_{n-1} + b_n\tan\alpha_n)\}/2 - \sum_{i=1}^{n}[m_i] \quad\cdots\cdots (36)$$

Therefore, the following relation can be obtained from equation (31),

$$\sum_{i=1}^{n}[M_i] = -\sum_{i=1}^{n}[Z_{i-1}\cdot\{\sin\theta_{i-1}\cdot(b_i + b_{i-1}) - \cos\theta_{i-1}\cdot(b_i\tan\alpha_i + b_{i-1}\tan\alpha_{i-1})\}/2] - \sum_{i=1}^{n}[m_i]$$
$$= -\sum_{i=1}^{n}[J] \quad\cdots\cdots (37)$$

Equation (37) clearly shows that the equation of moment equilibrium of the proposed method is essentially the same as the Spencer method(1973), though the sign of the equation becomes reverse.

Ⅳ Relation between inter-slice force, force and moment equilibrium in each method of slope stability analysis

The relation between each method of slope stability analysis and inter-slice force, force and moment equilibrium, is shown in Table 1. These methods are generally derived under force and/or moment equilibrium conditions with assumptions concerning the inter-slice force. The Morgenstern-Price and the Spencer (1973) methods are recognized as general limit equilibrium methods of slices and the factors of safety are computed by the conversing technique of the Newton-Raphson method. The method proposed in this paper is also a general limit equilibrium method of slices as simple as the Spencer method (1967, 1973).

Fredlund and Krahn compared several limit equilibrium methods of slices for slope stability analysis and showed the relation between inter-slice force, the force and moment equilibrium(1977). Figure 9 is an example of the section of a slice surface and Fig. 10 is the results of the analysis of this example. The value of the factor of safety F_m calculated by the moment equilibrium equation varied slightly with the change in the parameter λ, because X'_i, X'_{i+1} are involved only in p'. Furthermore, in the general slip surface, the factor of safety F_m is barely influenced by the inter-slice force, because the moments of P in each slice cancel each other totally and the value becomes relatively small. On the other hand, the value of the factor of safety F_f calculated by the force equilibrium equation varied greatly with a change of parameter λ, because the horizontal component of inter-slice force E is involved in the denominator of the slope stability equation (8).

Point A in Fig. 10, with $\lambda = 0$, results in an inter-slice force of zero. This gives the same value as the simplified Bishop method in the slip circle and the same value of the Nonveiller method in a general slip surface.

Point B gives the same result as the simplified Janbu method that has not been corrected. This value can be corrected by the correction factor or X'_i, X'_{i+1} calculated from the moment equilibrium equation and approaches the results of a rigorous solution (Fredlund and Krahn, 1977, Kawamoto, 1981)。

Table 1 Methods of slope stability analysis, force and moment equilibrium, and inter-slice force.

Methods of slope stability analysis	①	②	③	inter-slice force
Fellenius	○		○	
Simplified Janbu	○	○		
Simplified Bishop	○		○	
Nonveiller	○		○	
Spencer	○	○	○	Considered
Morgenstern-Price	○	○	○	Considered
The Proposed	○	○	○	Considered

①Vertical (perpendicular to the base of the slice) force equilibrium
②Horizontal (parallel to the base of the slice) force equilibrium
③Moment equilibrium
○･･･Satisfied

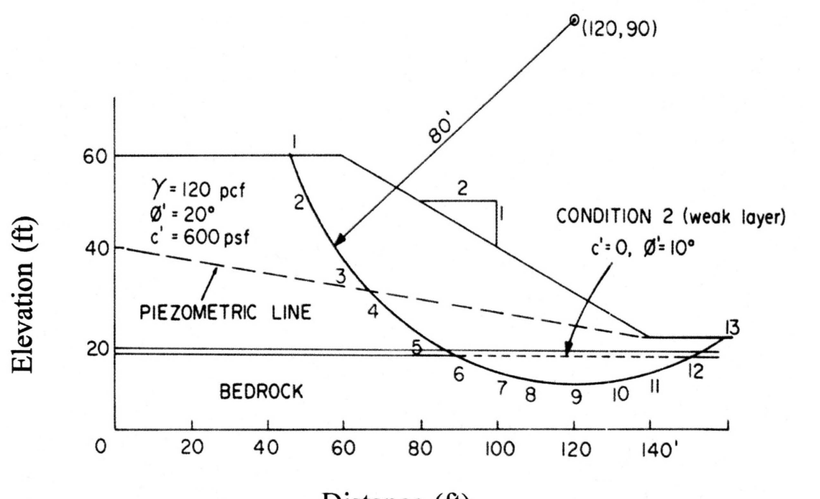


Fig. 9 An example of a slip surface (Fredlund and Krahn, 1977).

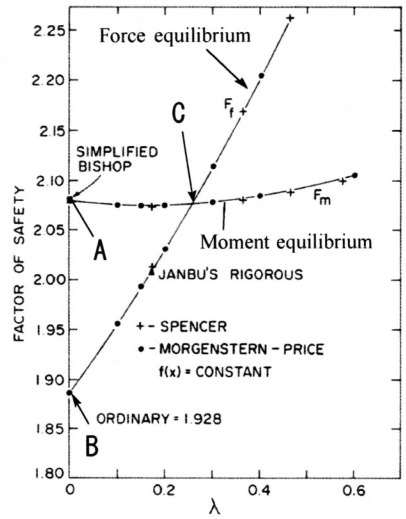
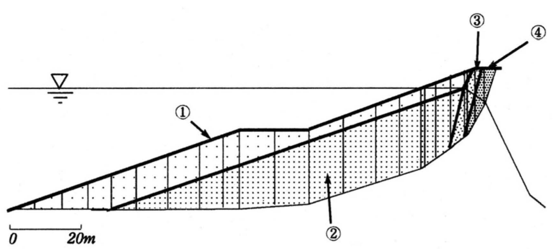


Fig. 10 Relation between the factor of safety, methods of slope stability analysis and λ (Fredlund and Krahn, 1977).

Point C in Fig.10 gives a value that satisfies the force and moment equilibrium conditions simultaneously, a so called rigorous solution. Because the change in the F_m value by λ is small, the simplified Bishop method and the Nonveiller method give results close to the rigorous solution in many cases, although inter-slice force is ignored, provided the common point of moment in the Nonveiller method is needed to be set as far as possible in the center of the general slip surface.

The Fellenius method, which is used in the analysis of general slip surface of landslide in Japan, generally gives smaller values than the rigorous solution. This value depends on the shape of slip circle and/or underground water level, whether smaller or larger than point C.

Figure 11 is an example of the slip surface of a fill dam and Fig. 12 shows the results of analysis of Fig.11 and the characteristics of force and moment function of three methods by parameter. The Spencer method (1973) and the proposed method give fundamentally the same value for factor of safety from the equations of force and moment equilibrium in the assumption $X'_i = E'_i \tan\theta_i$ (Furuya, 1986). The Morgenstern-Price method gives almost the same result as the Spencer method (1973) from the differential equations of force and moment equilibrium in the assumption $X'_i = \lambda \cdot f(x) \cdot E'_i$. This is similar to the assumption of the Spencer method (1973), even though the value of the results is slightly different because of the numerical solution technique (Kawamoto, 1981, Furuya, 1986, Kondo, 1997).



	①	②	③	④
γ_t (kN/m^3)	24.5	24.5	24.5	25.5
γ_{sat} (kN/m^3)	25.7	26.3	24.5	26.6
c' (kN/m^2)	26.46	4.9	0.0	14.7
ϕ' (degree)	36.0	35.0	33.0	22.0

Fig. 11 An example of the slip surface of a fill dam.

However, in the computation of the factor of safety from the moment equation, the characteristic of function F_m changes remarkably, depending on the position of the common point of moment (Furuya, 1986). The Spencer (1973) and the Morgenstern-Price methods satisfy the moment equilibrium conditions summing the inter-slice force of total sliding mass about the starting point of the slip surface as a common point of moment, point O_z (Fig. 8). On the other hand, the proposed method sums the inter-slice force of total sliding mass about point O_c as a common point of moment (Fig. 8). Figure 13 shows a slip surface and positions of center of moment as an example of computation. Figure 14 shows the

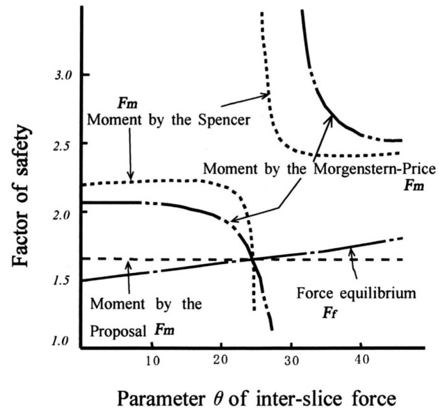


Fig. 12 Characteristics of force and moment function for three methods by parameter θ .

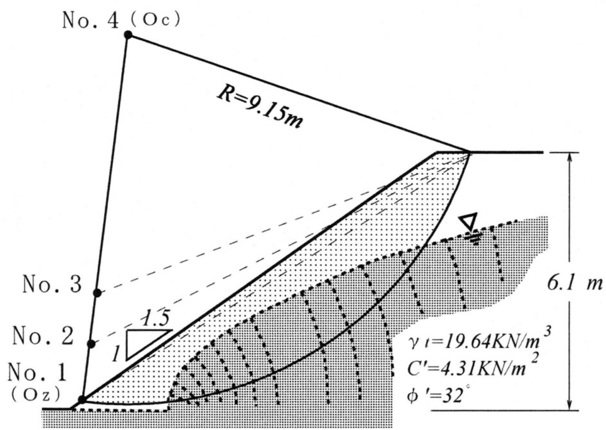


Fig. 13 An example of a slip surface (Lambe & Whitman, 1979)

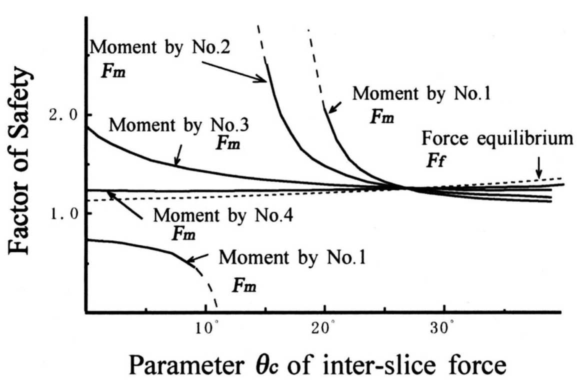


Fig. 14 Variation of the factor of safety that depend on the common point of moment

results of analysis of the slip surface in Fig. 13 and the characteristic of function F_m depending on the changes in the common point of moment. These figures show that the function F_m becomes almost simple straight line as approaches point O_c, however, that the function F_m becomes hyperbola about point O_z. In the hyperbola function, it is difficult to obtain simple result in the conversing computation of the Newton-Raphson method, if the initial value of θ_i is not proper. In this case, we need more optional work to find the proper initial value of θ_i, by writing a more complicated computing program. Even though we can obtain the results, the number of conversing computations increases as the curvature of a hyperbola increases. On the contrary, if the F_m function is close to a straight line, we can obtain results with a few conversing computations simply giving the initial value of θ_i. Therefore, we have an advantage in setting the point O_c as the common point of moment, when the moment of inter-slice forces is summed in the total sliding mass

(Furuya, 1997). The proposed method is more general for expression of inter-slice force than the Spencer method (1969), and more practical and advantageous in computation of moment in a total sliding mass than the Spencer (1973) and the Morgenstern- Price methods.

V Inter-slice force and side water pressure

Figures 15 and 16 show the example of comparative analysis of the section shown in Fig. 11 in the case of effective inter-slice forces X'_i and E'_i calculating side water pressure as statically determinate stress, and in the case of the total inter-slice forces X_i and E_i including side water pressure as statically indeterminate stress. These figures show that E_i is almost the same as $E'_i + P_{si}$, though X_i is slightly different from X'_i. Both computing methods gave the same factor of safety, as Ugai et al. (1985) also indicated a similar result. In the Ordinary method, thus, we can obtain more accurate results in the method that calculates side water pressure P_{si} as a statically determinate stress and ignores only X'_i and E'_i than in the method that ignores total inter-slice forces X_i and E_i.

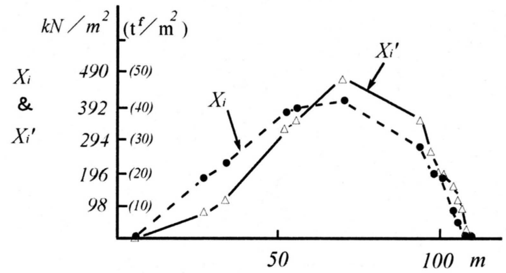


Fig. 15 Comparison of inter-slice force X_i and X'_i.

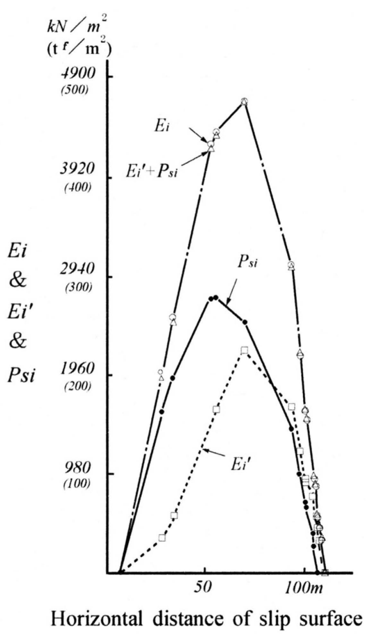


Fig. 16 Comparison of inter-slice force E_i and E'_i.

VI Computing method of pore water pressure by buoyancy (The buoyancy method)

In limit equilibrium methods of slices, there are two methods to compute the pore water pressure on each slice. One method individually calculates water pressures acting on the upper, both sides, and the base of the slice. The other is a method to calculate the buoyancy acting on the slice. The latter is simple and convenient, but, we need to modify the method for calculating buoyancy under the steady seepage flow conditions. Essentially the same results as the former can be obtained by the modified buoyancy method, which corrects the direction of the buoyancy vector by a hydraulic gradient (Furuya, 1985, 1996).

The buoyancy P_u becomes equal to the weight of water that is equivalent to the submerged space of the slice and acts vertically and upward as shown in Figs. 17 & 18. The buoyancy P_u acting on the slice under the steady seepage flow conditions is shown in Fig. 19. In this case, the buoyancy P_u changes the direction as shown in Fig. 20. If we assume the hydraulic gradient through each slice to be a straight line, the inclination to the vertical becomes equal to the hydraulic gradient ε as shown in Fig. 19.

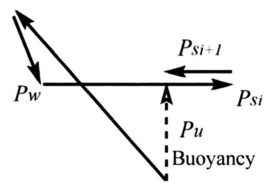


Fig. 17 Relation between water pressure and buoyancy P_u in a submerged slice .

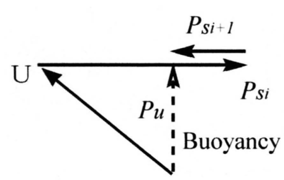


Fig. 18 Relation between water pressure and buoyancy P_u in a partially submerged slice .

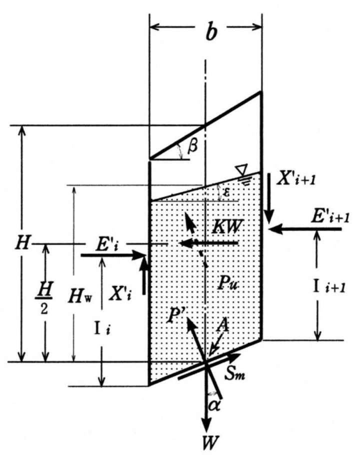


Fig. 19 Forces acting on a slice expressed by buoyancy .

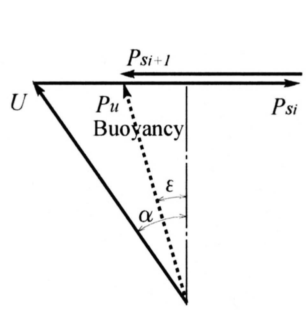


Fig. 20 Relation between water pressure and buoyancy P_u under the steady seepage flow conditions.

The buoyancy on the slice under the steady seepage flow condition acts vertically and upward and this force is equivalent to the water weight of the submerged space of the slice. At the same time, the force that is multiplied by tan ε acts on the slice horizontally. The hydraulic gradient ε is zero in submerged and partially submerged slices. Therefore, if we assemble the equation based on this idea, it is possible to express the buoyancy acting on the slices systematically under submerged, partially submerged, and steady seepage flow conditions.

Pore water pressures acting on the base and both sides of the slice should be calculated from the potential line in a strict sense, because pore water pressure under steady seepage flow conditions is different from the water depth at a point on the slip surface. However, the water depth is sometimes used as pore water pressure directly and conveniently, and the pore water pressure is excessively estimated when the hydraulic gradient changes rapidly. As a simple modified calculation, we can correct the pore water pressure by multiplying the water depth by $\cos^2\varepsilon$ in the slice, assuming a straight potential line (Fig. 21, Furuya, 1981). The side water pressure can also be modified by $\cos^2\varepsilon$, and buoyancy can be calculated by the modified direction. The equations for limit equilibrium methods of slices are similarly introduced by the polygon in Fig. 22 (Furuya, 1996).

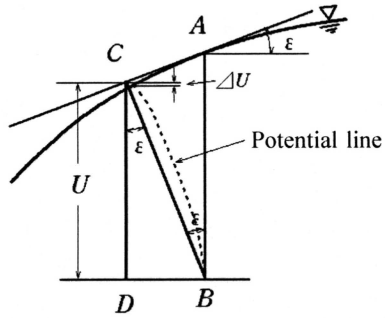


Fig. 21 A simple modified calculation of water pressure by a straight potential line

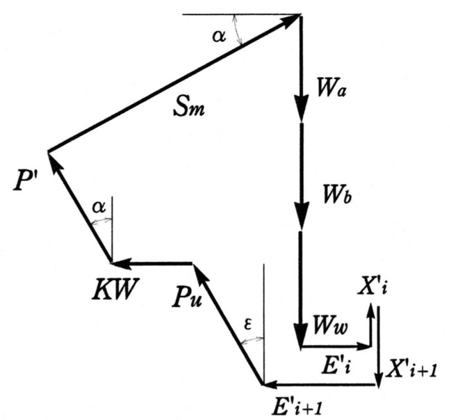


Fig. 22 Force polygon expressed by buoyancy under steady seepage flow conditions

King (1989) also described the steady seepage flow conditions in a assumption with a straight potential line (Figs. 23, 24) as indicating the water pressure Fs by seepage flow with buoyancy. The water pressure Fs is derived as follows:

$$Fs = \gamma_w \cdot b \cdot h \cdot \sin\theta x \quad \cdots\cdots \quad (38)$$

Where γ_w : unit weght of water

θx : same value of ε shown in Fig. 22

The resultant force becomes $\overline{d\,e}$ (Fig. 25) and gives essentially the same result as the proposal method that water pressure is modified by $\cos^2\varepsilon$ ($P_u = P'_u \cdot \cos^2\varepsilon$). As a result, The equation of the limit equilibrium methods of slices in the buoyancy method is introduced by the polygon in Fig. 22.

In addition, it has been considered that the conversing condition of inter-slice force, $\sum E = 0$ or $\sum E' = 0$ ($E = E' + Ps$) gives the same factor of safety in the general limit equilibrium method of slices. However, as King described in the discussion with Sarma (King, 1990), water pressure cancels in total sliding mass in case of a horizontal water surface. VIII

On the other hand, when seepage flow exists, the final results of the former computing method may be influenced by an error in water pressure balance because it is not canceled in the total sliding mass. Therefore, computing accuracy of the factor of safety certainly increases when water pressures are treated as statically determinate stresses in calculation of the water pressure on a slice. In calculation of water pressure by buoyancy, accuracy clearly increases when we consider the seepage force, if it is under steady seepage flow conditions.

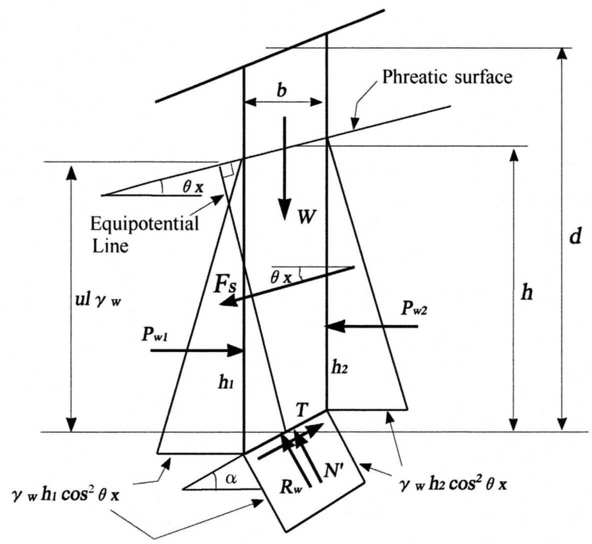


Fig. 23 Water pressure *Fs* by seepage flow and buoyancy (King, 1989).

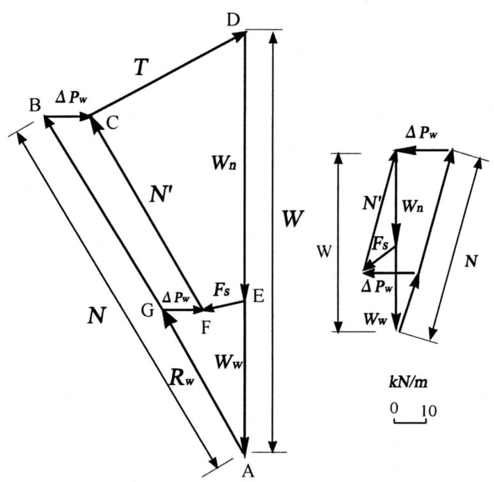


Fig. 24 Force polygon of a slice expressed by buoyancy and seepage force *Fs* (King, 1989).

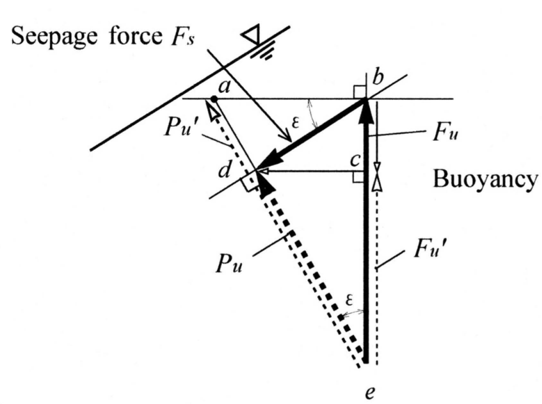


Fig. 25 Relation between buoyancy, seepage force *Fs* and P_u

The force equilibrium equation (24) of the proposed method can be rewritten by the buoyancy method as follows:

$$E'_{i+1} = \frac{[A]}{[D]} E'_i + \frac{[B]-[C]}{[D]} \quad \cdots\cdots (39)$$

where $[A] = \tan\theta_i \cdot \sin\alpha + \cos\alpha - \tan\phi'_m(\tan\theta_i \cdot \cos\alpha - \sin\alpha)$

$[B] = c'_m\, l + \tan\phi'_m\,(W_a + W_b)\cos\alpha - (W_a + W_b)\sin\alpha$

$[C] = (KW + P_u \cdot \sin\varepsilon)\cdot(\tan\phi'_m \sin\alpha + \cos\alpha)$

$[D] = \tan\theta_{i+1} \cdot \sin\alpha + \cos\alpha - \tan\phi'_m(\tan\theta_{i+1} \cdot \cos\alpha - \sin\alpha)$

The moment equilibrium equation (29) also can be rewritten by the buoyancy method as follows:

$$M = (x \cdot \tan\theta_{i+1} + y + H)\cdot E'_{i+1} - (x \cdot \tan\theta_i + y + H)\cdot E'_i + H \cdot KW/2 + H_w \cdot P_u \cdot \sin\varepsilon\ /2 \quad \cdots\cdots (40)$$

The factor of safety is computed from equations (38) and (40) by the Newton-Raphson method or linear reverse interpolation method.

Ⅶ Comparison of computation results by the buoyancy and the total pore water pressure methods

The section shown in Fig. 13 was analyzed by the proposed method of buoyancy described in chapter 6, with constant θ_i, θ_{i+1} and other methods. This is a case in which the hydraulic gradient rapidly changes in the slip surface and it is difficult to estimate exactly the pore water pressure from the water head in the slice. The pore water pressure, however, is calculated approximately from the water head in the slice corrected by $\cos^2\varepsilon$ as shown in Fig. 21.

The results of the slice method is affected by the number of slices in the slip surface, because of a slightly inexact acting point of water pressure and slice weight that are the center of the base or the upper. The shape of slip surface can be easily simulated correctly with more divided slices and the error of moment calculation also decreases, though input work and computation energy increases. The results of analysis are shown in Table 2. (a) of Table 2 is the results analyzed by Kawamoto (1981), with the slip surface divided into 26 slices by the total pore water pressure method. (b), (c), (d) are the results by the author. The calculation result is shown to three places of decimals for comparison, though it is enough to two places of decimals for practical purposes. The factor of safety by the Simplified Bishop method is 1.29, and 1.17 by the Ordinary method in the literature(Lambe & Whitman, 1979). These factors of safety shown by lambe & Whitman are a little larger than the results shown in Table 2 because the pore water pressure calculated by us is larger than the accurate value with the error of ΔU shown in Fig. 21. The proposed method with 26 slices by the buoyancy method gives the same result as the Spencer and the Morgenstern-Price methods with 28 slices by the total pore water pressure method. This indicates that the buoyancy method has less error in calculation of moment because of the same acting point of buoyancy to slice weight.

Table 2 Comparison of the results of slope stability analysis.

	(a) by Kawamoto(1981) 26 slices Total pore water	(b), 26 slices Total pore water	(c), 28 slices Total pore water	(d), 26 slices Buoyancy
Ordinary	1.123	1.125	1.127	1.126
Simplified Bishop	1.259	1.257	1.258	――
Nonveiller	――	――	――	1.250
Spencer	1.264	1.265	1.261	――
Morgenstern-Price	1.264	1.265	1.261	――
The Proposed	――	――	――	1.261

Conclusion

The Ordinary or Fellenius method has been the most fundamental and important method in the design of countermeasures for landslide or general slope stability analysis. This method gives a lower factor of safety than the general limit equilibrium methods of slices in many cases. However, this is improved partly by accurate computation of pore water pressure. The simplified Janbu method approximates the rigorous solution by a correction factor and the Janbu's rigorous method also gives good result which approaches the results of a rigorous solution in many cases. However, it is as workable as the general limit equilibrium method of slices. The Nonveiller method gives a factor of safety close to the rigorous solution in many cases, even though inter-slice force is ignored, if we set the common point of moment as close to the center point of slip surface as possible.

Nowadays, we can utilize high performance personal computers conveniently and easily compute factors of safety automatically by the general limit equilibrium methods of slices, such as the Morgenstern-Price method, the Spencer methods, and the method proposed in this study. In computation of the general limit equilibrium method of slices, we have an advantage in setting the common point of moment as near the center of slip surface as possible rather than setting it at the beginning point of the slip surface. The proposed method is simpler in expression of equations and also practical computation for this reason. This is very important for extension to three-dimensional analysis and/or application to geologically complicated slip surfaces.

In addition, a method of how to determine the factor $f(x)$ or θ_i of inter-slice force reasonably needs to be developed and this is a topic for on going research.

We can calculate simply and accurately pore water pressure by the seepage force with buoyancy under hydrostatic pressure and steady seepage flow conditions with the assumption that hydraulic line is straight in a slice, if the hydraulic gradient is not too large. When the hydraulic gradient rapidly changes, the width of the slice needs to be divided into smaller to increase calculating accuracy.

References

Bishop, A.W.(1955) : The use of the slip circle in the stability analysis of slopes. Geotechnique, 5, p.7- 17

Fredlund, D.G., Krahn,J. (1977) : Comparison of slope stability methods of analysis. Can.Geotech.J., 14, p.429-439

Fredlund, D.G., Krahn, J. and Pufahl, D.E.(1981) : The relationship between limit equilibrium slope stability methods. Proc.10th. I.C.S.M.F.E. Vol.Ⅲ, p.409-416

Furuya, T.(1981) : Shear strength of tertiary mudstone and clay sampled at Narao landslide district and examination of shear strength parameters by slope stability analysis. Technical report of the National Research Institute of Agricultural Engineering, Series C22, p.41-52 (in Japanese)

Furuya, T.(1985) : A method to calculate water pressure on a slice as a buoyancy in steady seepage flow condition. Proceeding of the Japanese society of irrigation, drainage and reclamation engineering, p.422-423 (in Japanese)

Furuya, T.(1985) : Slope stability analysis utilizing the principles of limit equilibrium. Transactions of the Japanese society of irrigation, drainage and reclamation engineering, 120, p.311-316 (in Japanese)

Furuya, T.(1986) : Slope stability analysis utilizing the principles of limit equilibrium (Ⅱ). Transactions of the Japanese society of irrigation, drainage and reclamation engineering, 125, p.9-17 (in Japanese)

Furuya, T.(1996) : A method to calculate water pressure on a slice as a buoyancy in steady seepage flow condition and a general limit equilibrium method of slices. Journal of Japan landslide society, 33 (1), p.9-14 (in Japanese)

Furuya, T.(1997) : Consideration on the Spencer's general limit equilibrium method of slices. Journal of Japan landslide society, 34(2), p.9-14 (in Japanese)

Janbu, N.(1954) : Application of Composite Slip Surfaces for Stability Analysis. Proc. European Conf. on Stability of Earth Slopes, 3, p.43-49

Kawamoto, O.(1981) : Examination of limiting equilibrium methods considering inter-slice forces and application to stability analysis of landslide. Technical report of the National Research Institute of Agricultural Engineering, Series

C22, p.53-67 (in Japanese)
King, G.J.W.(1989) : Revision of effective stress method of slices. Geotechnique, 39, p.497-502
King, G.J.W(1990) : [DISCUSSION] Revision of effective-stress method of slices. Geotechnique, 40, p.651-654
Kondo, K., Hayashi, S.(1997): Similarity and generality of the Morgenstern-Price method and Spencer method, Journal of Japan landslide society. 34(1), p.15-23 (in Japanese)
Lambe, T.W., Whitman, R.V.(1979): Soil Mechanics, SI Version. John Wiley & Sons, p.359
Morgenstern, N.R. and Price,V.E.(1965): The Analysis of the stability of general slip surfaces. Geotechnique, 15, p.79-93
Nonveiller, E.(1965): The Stability Analysis of Slopes with a Slip Surface of General Shape. Proc.6th. I.C.S.M.F.E. Vol.Ⅱ, p.522-525
Spencer, E.(1967): A method of analysis of the stability of embankments assuming parallel inter-slice forces. Geotechnique, 17, p.11-26
Spencer, E.(1969) : Circular logarithmic spiral slip surfaces. A.S.C.E., 95, SM1, p.227-234
Spencer, E.(1973) : Thrust line criterion in embankment stability analysis. Geotechnique, 23, p.311-316
Sarma, S.K.(1973) : Stability analysis of embankments and slopes. Geotechnique, 23, p.423-433
Ugai, K., Hosobori,K.(1985)：Considerations of side water forces in the slice method. Soils and Foundations, 33(4), p.39-42 (in Japanese)
Yamagami, T., Ueda, Y. (1982) : Some considerations on stability analysis of submerged slopes. Soils and Foundations, 30(12), p.19-26 (in Japanese)

斜面安定解析のための極限平衡分割法の比較と評価

— モーメント平衡式の改良と定常浸透状態における水中重量法 —

古谷　保

適 用

日本の第三紀層泥岩や破砕帯等の地すべり地帯においては，地すべりや斜面崩壊がたびたび発生するにもかかわらず，古くから農地として利用されてきたところが多い。このため斜面安定問題は農地保全においても重要な課題であり，地すべりや斜面崩壊の安定性の検討に，極限平衡分割法による斜面安定解析式が一般に用いられている。この方法には多くの方法が提案されてきたが，簡便法や簡易Bishop法，簡易Janbu法，Nonveiller法，Morgenstern-Price法，Spencer法(1967, 1973)等が広く利用されてきた。本論では，これらの方法を要約するとともに，筆者の提案する一般分割法と定常浸透状態における水中重量法を紹介し，それらの比較検討を行っている。

簡便法は地すべり対策工の設計において最も基本となる式であり，いわゆる厳密解と比較していくぶん低い安全率が得られるが，間隙水圧の計算を精確に行えば精度は改善される。簡易Janbu法は補正係数により一般に厳密解に近づく。また厳密Janbu法の場合も同様であるが，労力的には一般分割法とあまり変わらない。Nonveiller法は不静定内力を無視しても，多くの場合において厳密解にある程度近い結果が得られるが，モーメントの中心点の取り方として，極力，すべり面の中心に近い点を選ぶ必要がある。

今日ではパーソナルコンピュータが簡便に利用できるので，一般分割法による厳密計算も容易に行うことができる。一般分割法では，Morgenstern-Price法やSpencer法(1973)のようにモーメントの中心をすべり面の始点とするより，提案法のようにすべり面の中心に近い点として式を組み立てる方が収束計算が簡単になるので有利である。提案法は，Morgenstern-Price法やSpencer法(1973)と比較して，同程度の厳密解をより簡単な収束計算で解くことが出来る。また水中重量法は，スライス内の動水勾配を直線近似して浮力ベクトルの方向を修正すれば，定常浸透状態にも適用でき，分割法の式と計算が一層単純化される。

なお、本論は「安定解析式の検討」（シンポジウム「地すべりに関わるモデル解析と実際」，地すべり学会, 1998）を基本にして、英文で纏め直されたものであることを付記する。

キーワード：斜面安定，極限平衡，分割法，間隙水圧，浮力